

## Thermal nature of de Sitter spacetime and spontaneous excitation of atoms

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**ABSTRACT:** We consider, in de Sitter spacetime, both freely falling and static two-level atoms in interaction with a conformally coupled massless scalar field in the de Sitter-invariant vacuum, and separately calculate the contributions of vacuum fluctuations and radiation reaction to the atom's spontaneous excitation rate. We find that spontaneous excitations occur even for the freely falling atom as if there is a thermal bath of radiation at the Gibbons-Hawking temperature and we thus recover, in a different physical context, the results of Gibbons and Hawking that reveals the thermal nature of de Sitter spacetime. Similarly, for the case of the static atom, our results show that the atom also perceives a thermal bath which now arises as a result of the intrinsic thermal nature of de Sitter spacetime and the Unruh effect associated with the inherent acceleration of the atom.

**KEYWORDS:** Models of Quantum Gravity, Nonperturbative Effects.

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**1. Introduction**

Spontaneous emission is one of the most interesting problems in the interaction of atoms with quantum fields and so far mechanisms such as vacuum fluctuations [1, 2], radiation reaction [3], or a combination of them [4] have been put forward to explain why spontaneous emission occurs. The controversy arises because of the freedom in choices of ordering of commuting operators of atom and field in a Heisenberg picture approach to the problem and was resolved when Dalibard, Dupont-Roc and CohenTannoudji(DDC) argued in Ref.[5] and Ref.[6] that there exists a symmetric operator ordering that renders the distinct contributions of vacuum fluctuations and radiation reaction to the rate of change of an atomic observable separately Hermitian. If one demands such an ordering, each contribution can possess an independent physical meaning. The DDC formalism resolves the problem of stability for ground-state atoms when only radiation reaction is considered and the problem of “spontaneous absorption” of atoms in vacuum when only vacuum fluctuations are taken into account. Using this formalism one can show that for ground-state atoms, the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean excitation energy cancel exactly and this cancellation forbids any transitions from the ground state and thus ensures atom’s stability. While for any initial excited state, the rate of change of atomic energy acquires equal contributions from vacuum fluctuations and from radiation reaction.

Recently, there has been a great deal of interest in the application of the DDC formalism to accelerated atoms and those in the background of a black hole [7–13]. These studies reveal intriguing relationships between spontaneous excitation of an atom and the Unruh effect as well as the Hawking radiation. In a flat spacetime, the spontaneous excitation of a uniformly accelerated atom in interaction with vacuum scalar [7, 8, 11] and electromagnetic fields [9, 10] has been studied. It is found that for a ground state atom in uniformly accelerated motion through the Minkowski vacuum, there is no longer perfect

balance between vacuum fluctuations and radiation reaction. As a result, the spontaneous excitation rate of the atom is nonzero and furthermore the rate is exactly what one would obtain assuming the existence of a thermal bath at the Unruh temperature. Inspired by an equivalence principle-type argument, the spontaneous excitation rate of atoms in interaction with a massless scalar field in an interesting kind of curved spacetimes, i.e., the curved background of a black hole, has recently been studied [12, 13], in both the Hartle-Hawking vacuum and Unruh vacuum. The results obtained may be considered as providing a different approach to derivation of the Hawking effect, since they show that a static atom in the exterior of a black hole would spontaneously excite as if immersed in a thermal bath of Hawking radiation.

As natural step forward, we are interested, in the present paper, in the spontaneous excitation of atoms in yet another kind of special curved spacetime—de Sitter spacetime. De Sitter spacetime, being maximally symmetric, enjoys an important status among the curved spacetimes similar to that of Minkowski spacetime, and more importantly, it has attracted a surge of renewed interest in recent years for the following reason: First, recent observations, together with the theory of inflation, suggest that our universe may approach de Sitter geometries in both the far past and the far future, and second, there may exist a holographic duality between quantum gravity on de Sitter spacetime and a conformal field theory living on the boundary identified with the timelike infinity of de Sitter spacetime [14]. Therefore, it is certainly of interest to examine the spontaneous excitation of atoms in this spacetime and this is what we plan to do in the present paper. Using the DDC formalism, we will calculate the spontaneous excitation rate of both a freely falling atom and a static one with an inherent acceleration in interaction with vacuum fluctuations of quantized massless conformally coupled scalar fields in de Sitter spacetime. Let us note that the quantization of scalar fields in this spacetime has been extensively studied in the literature [15–24].

When vacuum fluctuations are concerned in a curved spacetime, one first has to specify the vacuum states. The vacuum states in de Sitter spacetime can be classified into two categories: one is the de Sitter-invariant states, the other states are those which break de Sitter invariance [21]. Generally, the de Sitter-invariant vacuum, whose status in de Sitter space is just like Minkowski vacuum in the flat space, is deemed to be a natural vacuum. So, we will investigate the spontaneous excitation of atoms in interaction with a conformally coupled massless scalar field in the de Sitter-invariant vacuum. We will show that for an atom moving on a timelike geodesic (freely falling), the spontaneous excitation rate is what one would expect if the atom were in a thermal bath of radiation at the Gibbons-Hawking temperature [25]. While for a static atom in the de Sitter-invariant vacuum, we find that it also may spontaneously excite as if immersed in a thermal bath at a temperature equal to the square root of the sum of the squared Gibbons-Hawking temperature and the squared Unruh temperature associated with the inherent acceleration of the atom.

It is worth pointing out that the difference between analyzing the spontaneous excitation of atoms using the DDC formalism as we do in the current paper and similar previous calculations of the response of model detectors in de Sitter spacetime [15, 25] lies in that our discussions provide a physically appealing interpretation of the thermal response of the

detector, i.e., a transparent illustration for why the detector clicks, since the spontaneous excitation of the atoms can be considered as the actual physical process that is actually taking place inside a model detector revealing the thermal nature of de Sitter spacetime.

## 2. The general formalism

We consider a pointlike two-level atom in interaction with a conformally coupled massless scalar field in de Sitter spacetime and assume that the atom has a stationary trajectory  $x(\tau)$ , where  $\tau$  denotes the proper time on the trajectory. This stationary trajectory guarantees that the atom has stationary states,  $|-\rangle$  and  $|+\rangle$ , with energy  $-\frac{1}{2}\omega_0$  and  $\frac{1}{2}\omega_0$ . The atom's Hamiltonian with respect to its proper time  $\tau$  can be written as [26]

$$H_A(\tau) = \omega_0 R_3(\tau) , \tag{2.1}$$

where  $R_3(0) = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$ . The free Hamiltonian of the quantum scalar field is

$$H_F(\tau) = \int d^3k \omega_{\vec{k}} a_{\vec{k}}^\dagger a_{\vec{k}} \frac{dt}{d\tau} , \tag{2.2}$$

where  $a_{\vec{k}}^\dagger$  and  $a_{\vec{k}}$  denote the creation and annihilation operators with momentum  $\vec{k}$ . The Hamiltonian that describes the interaction between the atom and the quantum field is given by [7]

$$H_I(\tau) = \mu R_2(\tau) \phi(x(\tau)) . \tag{2.3}$$

Here  $\mu$  is a coupling constant which we assume to be small,  $R_2(0) = \frac{1}{2}i[R_-(0) - R_+(0)]$ , where  $R_+(0) = |+\rangle\langle-|$  and  $R_-(0) = |-\rangle\langle+|$ .  $\phi(x)$  is the scalar field operator in de Sitter spacetime and it satisfies the wave equation

$$(\nabla_\mu \nabla^\mu + m^2 + \xi R)\phi = 0 , \tag{2.4}$$

where  $m$  is the mass of the scalar field,  $\xi$  is a coupling constant and  $R$  is the scalar curvature. The coupling is effective only on the trajectory of the atom.

Then we can write down the Heisenberg equations of motion for the dynamical variables of the atom and field from the Hamiltonian  $H = H_A + H_F + H_I$ . The solutions of the equations of motion can be split into the two parts: a free part, which is present even in the absence of the coupling, and a source part, which is caused by the interaction of the atom and field. We assume that the initial state of the field is the de Sitter-invariant vacuum (also known as Euclidean or Bunch-Davies vacuum [16])  $|0\rangle$ , and the atom is prepared in the state  $|a\rangle$ , which may be  $|+\rangle$  or  $|-\rangle$ . Choosing a symmetric ordering between the atom and field variables, we can separate the two contributions of vacuum fluctuations and radiation reaction to the rate of change of  $\langle H_A \rangle$  ( cf. Refs. [7, 5, 6] ),

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x, x') \frac{d}{d\tau} \chi^A(\tau, \tau') , \tag{2.5}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x, x') \frac{d}{d\tau} C^A(\tau, \tau') , \tag{2.6}$$

where  $|\rangle = |a, 0\rangle$  represents the atom in the state  $|a\rangle$  and the field in the de Sitter-invariant vacuum state  $|0\rangle$ . They are expressed in terms of the statistical functions of the free part of the atom's variable,  $R_2^f$

$$C^A(\tau, \tau') = \frac{1}{2} \langle a | \{R_2^f(\tau), R_2^f(\tau')\} | a \rangle, \quad (2.7)$$

$$\chi^A(\tau, \tau') = \frac{1}{2} \langle a | [R_2^f(\tau), R_2^f(\tau')] | a \rangle, \quad (2.8)$$

and those of the field's,  $\phi^f$ ,

$$C^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{\phi^f(x(\tau)), \phi^f(x(\tau'))\} | 0 \rangle, \quad (2.9)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | [\phi^f(x(\tau)), \phi^f(x(\tau'))] | 0 \rangle. \quad (2.10)$$

$C^F$  ( $C^A$ ) is called the symmetric correlation function of the field (atom),  $\chi^F$  ( $\chi^A$ ) its linear susceptibility. The explicit forms of the statistical functions of the atom are given by

$$C^A(\tau, \tau') = \frac{1}{2} \sum_b |\langle a | R_2^f(0) | b \rangle|^2 \left( e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')} \right), \quad (2.11)$$

$$\chi^A(\tau, \tau') = \frac{1}{2} \sum_b |\langle a | R_2^f(0) | b \rangle|^2 \left( e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')} \right), \quad (2.12)$$

where  $\omega_{ab} = \omega_a - \omega_b$  and the sum extends over a complete set of atomic states.

### 3. Spontaneous excitation of a freely falling atom in de Sitter spacetime

In this Section we will consider a freely moving atom interacting with a conformally coupled massless scalar field in de Sitter spacetime. As is well known, different coordinates systems can be used to parameterize de Sitter spacetime [15]. The rate of change of the atomic energy is a scalar and should be independent of the coordinates. Here we choose to work in the global coordinate system, in which the line element is expressed as

$$ds^2 = dt^2 - \alpha^2 \cosh^2(t/\alpha) [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)]. \quad (3.1)$$

Here  $\alpha = 3^{1/2} \Lambda^{-1/2}$ , where  $\Lambda$  is the cosmological constant, and the scalar curvature  $R = 12\alpha^{-2}$ . The canonical quantization of a massive scalar field with this metric has been dealt with in Refs. [15, 18, 20–22]. In coordinates (3.1), the wave equation (2.4) for a massive scalar field becomes

$$\left[ \frac{1}{\cosh^3 t/\alpha} \frac{\partial}{\partial t} \left( \cosh^3 \frac{t}{\alpha} \frac{\partial}{\partial t} \right) - \frac{\Delta}{\alpha^2 \cosh^2 t/\alpha} + m^2 + \xi R \right] \phi = 0, \quad (3.2)$$

where the Laplacian

$$\Delta = \frac{1}{\sin^2 \chi} \left[ \frac{\partial}{\partial \chi} \left( \sin^2 \chi \frac{\partial}{\partial \chi} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right]. \quad (3.3)$$

From (3.2) one can get the eigenmodes, and define a de Sitter-invariant vacuum. Then the Wightman function can be written as [22]

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi\alpha^2} \frac{\frac{1}{4} - \nu^2}{\cos \pi\nu} F\left(\frac{3}{2} + \nu, \frac{3}{2} - \nu; 2; \frac{1 - Z(x, x')}{2}\right), \quad (3.4)$$

where  $F$  is a hypergeometric function, and

$$\begin{aligned} Z(x, x') &= \sinh \frac{t}{\alpha} \sinh \frac{t'}{\alpha} - \cosh \frac{t}{\alpha} \cosh \frac{t'}{\alpha} \cos \Omega \\ \cos \Omega &= \cos \chi \cos \chi' + \sin \chi \sin \chi' [\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi')]', \\ \nu &= \left[ \frac{9}{4} - \frac{12}{R}(m^2 + \xi R) \right]^{1/2}. \end{aligned} \quad (3.5)$$

In the massless, conformally coupled limit, for a freely falling atom, the Wightman function becomes

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi^2\alpha^2 \sinh^2(\frac{\tau - \tau'}{2\alpha} - i\varepsilon)}. \quad (3.6)$$

Then the statistical functions of the field, (2.9) and (2.10), can be obtained

$$C^F(x(\tau), x(\tau')) = -\frac{1}{32\pi^2\alpha^2} \left[ \frac{1}{\sinh^2(\frac{\tau - \tau'}{2\alpha} - i\varepsilon)} + \frac{1}{\sinh^2(\frac{\tau - \tau'}{2\alpha} + i\varepsilon)} \right], \quad (3.7)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{i}{4\pi \cos(\frac{i(\tau - \tau')}{2\alpha})} \delta'(\tau - \tau'). \quad (3.8)$$

With a substitution  $u = \tau - \tau'$ , the contributions of vacuum fluctuations (2.5) and radiation reaction (2.6) to the rate of change of the atomic energy read

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = -\frac{\mu^2}{32\pi^2\alpha^2} \sum_b \omega_{ab} |\langle a | R_2^f(0) | b \rangle|^2 \int_{-\infty}^{+\infty} du \left[ \frac{1}{\sin^2(\frac{i u}{2\alpha} + \varepsilon)} + \frac{1}{\sin^2(\frac{i u}{2\alpha} - \varepsilon)} \right] e^{i\omega_{ab}u} \quad (3.9)$$

and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = -\frac{i\mu^2}{4\pi} \sum_b \omega_{ab} |\langle a | R_2^f(0) | b \rangle|^2 \int_{-\infty}^{+\infty} du \frac{e^{i\omega_{ab}u}}{\cos(\frac{i u}{2\alpha})} \delta'(u), \quad (3.10)$$

where we have extended the range of integration to infinity for sufficiently long times  $\tau - \tau_0$ . With the help of residue theorem, we can evaluate the integrals to get

$$\begin{aligned} \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} &= -\frac{\mu^2}{4\pi} \left[ \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{2}{e^{2\pi\alpha\omega_{ab}} - 1} \right) \right. \\ &\quad \left. - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{2}{e^{2\pi\alpha|\omega_{ab}|} - 1} \right) \right] \end{aligned} \quad (3.11)$$

for the contributions of vacuum fluctuations to the rate of change of atomic energy, and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = -\frac{\mu^2}{4\pi} \left( \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \right) \quad (3.12)$$

for that of radiation reaction. Comparison with the case of an inertial atom in the Minkowski vacuum [7] shows that the contribution of radiation reaction to the rate of change of the atomic energy is the same as that in the flat spacetime, and thus is independent of the spacetime curvature. But the contribution of vacuum fluctuations is modified by the appearance of a thermal like term. Adding up the contributions of the vacuum fluctuations (3.11) and radiation reaction (3.12) we arrive at the total rate of change of the atomic energy:

$$\begin{aligned} \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = & -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{1}{e^{2\pi\alpha\omega_{ab}} - 1} \right) \right. \\ & \left. - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \frac{1}{e^{2\pi\alpha|\omega_{ab}|} - 1} \right]. \end{aligned} \quad (3.13)$$

For an atom in its ground state in the de Sitter-invariant vacuum, there is a positive contribution. So the atom spontaneously excites, just as if it were in a bath of blackbody radiation at the temperature  $T = 1/2\pi\alpha$ , which is exactly the temperature found by Gibbons and Hawking [25] by examining the response of a freely moving particle-detector in de Sitter spacetime. We therefore recover, in a different physical context, the results of Gibbons and Hawking that reveals the thermal nature of de Sitter spacetime [25]. It should be pointed out that since the cosmological constant is a very small number, so the temperature  $T$  is insignificant in terms of the experimental observation.

#### 4. Spontaneous excitation of a static atom in de Sitter spacetime

Now, we will calculate the spontaneous excitation rate of a static atom in de Sitter spacetime interacting with a conformally coupled massless scalar field in the de Sitter-invariant vacuum. For this purpose, it is convenient to work in the static coordinate system in which the line element is written as

$$ds^2 = \left( 1 - \frac{r^2}{\alpha^2} \right) d\tilde{t}^2 - \left( 1 - \frac{r^2}{\alpha^2} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (4.1)$$

This metric possesses a coordinate singularity of the type of an event horizon at  $r = \alpha$ . The coordinates  $(\tilde{t}, r, \theta, \varphi)$  only cover part of de Sitter spacetime, just like the Rindler wedge. For an atom at rest in the static coordinates system, its inherent acceleration is

$$a = \frac{r}{\alpha^2} \left( 1 - \frac{r^2}{\alpha^2} \right)^{-1/2}. \quad (4.2)$$

The static coordinates are related to the global coordinates by

$$r = \alpha \cosh(t/\alpha) \sin \chi, \quad \tanh(\tilde{t}/\alpha) = \tanh(t/\alpha) \sec \chi. \quad (4.3)$$

It should be noted that the worldlines of observers in the global and static coordinates coincide at  $r = 0$  and  $\chi = 0$  and an atom at rest in the static coordinates with  $r \neq 0$  will be accelerated with respect to an atom at rest in the global coordinates with  $\chi = 0$ . In

the static de Sitter metric (4.1), one can obtain a complete set of mode solutions of (2.4) [23, 24], and chooses a de Sitter-invariant vacuum. Then the Wightman function for a massless conformally coupled scalar field is given by [27, 28]

$$G^+(x(\tau), x(\tau')) = -\frac{1}{8\pi^2\alpha^2} \frac{\cosh(\frac{r^*}{\alpha}) \cosh(\frac{r^{*'}}{\alpha})}{\cosh(\frac{\tilde{t}-\tilde{t}'}{\alpha} - i\varepsilon) - \cosh(\frac{r^*-r^{*'}}{\alpha})}, \quad (4.4)$$

where

$$r^* = \frac{\alpha}{2} \ln \frac{\alpha + r}{\alpha - r}. \quad (4.5)$$

For a static atom, Eq. (4.4) becomes

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi^2\kappa^2 \sinh^2(\frac{\tau-\tau'}{2\kappa} - i\varepsilon)}, \quad (4.6)$$

where  $\kappa = \alpha\sqrt{g_{00}}$ , and we have used the definition

$$\Delta\tau = \sqrt{g_{00}}\Delta\tilde{t}. \quad (4.7)$$

One can then show that

$$C^F(x(\tau), x(\tau')) = -\frac{1}{32\pi^2\kappa^2} \left[ \frac{1}{\sinh^2(\frac{\tau-\tau'}{2\kappa} - i\varepsilon)} + \frac{1}{\sinh^2(\frac{\tau-\tau'}{2\kappa} + i\varepsilon)} \right], \quad (4.8)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{i}{4\pi \cos(\frac{i(\tau-\tau')}{2\kappa})} \delta'(\tau - \tau'). \quad (4.9)$$

With the statistical functions given, we can compute the contributions of the vacuum fluctuations and radiation reaction to the rate of change of the atomic energy to get

$$\begin{aligned} \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = & -\frac{\mu^2}{4\pi} \left[ \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{2}{e^{2\pi\kappa\omega_{ab}} - 1} \right) \right. \\ & \left. - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{2}{e^{2\pi\kappa|\omega_{ab}} - 1} \right) \right]. \quad (4.10) \end{aligned}$$

and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = -\frac{\mu^2}{4\pi} \left( \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \right). \quad (4.11)$$

Adding up the contributions of the vacuum fluctuations (4.10) and radiation reaction (4.11) we obtain the total rate of change of the atomic energy:

$$\begin{aligned} \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = & -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \left( 1 + \frac{1}{e^{2\pi\kappa\omega_{ab}} - 1} \right) \right. \\ & \left. - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2^f(0) | b \rangle|^2 \frac{1}{e^{2\pi\kappa|\omega_{ab}} - 1} \right]. \quad (4.12) \end{aligned}$$



The above calculations show that the contribution of radiation reaction to the spontaneous excitation rate is independent of the spacetime curvature and the acceleration. From Eq. (4.12) we can see that for a static atom with an inherent acceleration in its ground state in the de Sitter-invariant vacuum, there is a positive contribution to the spontaneous excitation rate. Thus transitions from ground state to the excited states become possible, and they occur just as if the atoms were immersed in a thermal bath at the temperature

$$T = \frac{1}{2\pi\kappa} = \frac{1}{2\pi\sqrt{g_{00}\alpha}}. \quad (4.13)$$

It is interesting to note that the above temperature can be written in the form

$$T^2 = \left(\frac{1}{2\pi\alpha}\right)^2 + \left(\frac{a}{2\pi}\right)^2 = T_{GH}^2 + T_U^2. \quad (4.14)$$

The first spacetime curvature dependent term in the above equation is the squared Gibbons-Hawking temperature of de Sitter spacetime, which is the temperature of thermal radiation as perceived by a freely falling atom, and the second acceleration dependent term is the squared Unruh temperature which arises as a result of the Unruh effect. Thus, the thermal radiation as felt by a static atom in de Sitter spacetime is a combination of two different effects. One arises from the thermal nature of de Sitter spacetime itself and is characterized by the Gibbons-Hawking temperature and the other from the acceleration induced thermal effect characterized by the Unruh temperature. This is quite similar to what happens for a static atom outside a black hole, where one finds that both the Hawking effect and the Unruh effect contribute to the bath of thermal radiation encountered by the atom [12, 13], although the detailed relationship is different. Relation Eq. (4.14) agrees with the result obtained in other different physical contexts [29, 30]. Let us note that  $T$  diverges as  $r$  approaches  $\alpha$  as a consequence of the blowup of  $T_U$ . The reason is that to hold the atom static at the event horizon, an infinite inherent acceleration is needed.

## 5. Summary

We have studied, using the DDC formalism, the spontaneous excitation of a two-level atom interacting with a conformally coupled massless scalar field in the de Sitter-invariant vacuum in de Sitter spacetime, and separately calculated the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy. Both the case of a freely falling atom and that of a static atom have been considered.

Remarkably, for a freely falling atom, we find a nonzero excitation rate, which is exactly what one would obtain if there is a bath of thermal radiation at the Gibbons-Hawking temperature. We therefore recover, in a different physical context, the results of Gibbons and Hawking that reveals the thermal nature of de Sitter spacetime [25].

For a static atom, our results show that the atom also perceives a thermal bath of radiation which is now a combined result of the Gibbons-Hawking effect of de Sitter spacetime and the Unruh effect associated with the inherent acceleration the atom must have in order to be static. An interesting feature in contrast to the case of a static atom outside

a black hole [12, 13] is that the temperature of thermal bath as perceived by the static atom in de Sitter spacetime is the square root of the sum of the squared Gibbons-Hawking temperature and the squared Unruh temperature associated with the inherent acceleration of the atom.

Finally, It should be pointed out that we have only considered the de Sitter invariant Bunch-Davies states, and it, therefore, remains interesting to see whether the thermal response will still be present when the vacuum states are replaced by a wider class of states as those considered in Refs. [31, 32].

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